Estimation of Traffic Capacity at a Sightseeing Area
by using Network Equilibrium Theory

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Abstract

This paper aims to propose a method which can evaluate the capacity of a sightseeing area. At first, we defined the capacity as the maximum level of visitors which can be guaranteed a suitable sightseeing activity. Secondly, an evaluation method to estimate the capacity of a sightseeing site is proposed. This method is based on the stochastic user equilibrium assignment simulation technique with elastic destination choice because a traveler may change his destination and route according to the congestion level en route and at his original destination site. Finally, this method is applied to the Aso area and its applicability is examined.

Introduction

In Japan, there are some sightseeing areas where environmental disruption and traffic congestion occur with an increase of sightseeing traffic demand. These phenomena may decrease the attraction of themselves. To maintain comfortable sightseeing for visitors, some measures like entrance control and route guidance may be needed. At that time, we need to know the capacity of a sightseeing site in advance. In this paper, we define the capacity of a sightseeing area as the maximum level of entrance demand which a site can accept while guaranteeing a suitable sightseeing activity to visitors. The second aim of this paper is to propose a mathematical tool to evaluate this maximum level of entrance demand. Finally, we

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apply this method to a national park area and evaluate its applicability.

This paper is organized in the following manner. Section 2 briefly investigates some characteristics of sightseeing by car and defines the capacity of a sightseeing area considering these characteristics of sightseeing. In Section 3, a mathematical model to evaluate the sightseeing capacity, and unknown parameters included in this model are proposed in Section 4. In Section 5, this method is applied to Aso national park area to evaluate the sightseeing capacity of this area.

Capacity of a Sightseeing Area

As an index defining the maximum level of activity in an urban area, network capacity is generally used. Network capacity is defined as "the maximum level of traffic demand which a traffic network can accept physically and/or economically", i.e., the total traffic demand with a fixed OD pattern when at least one OD pair’s flow is not able to reach its destination because traffic flow on some links exceed their capacity as total traffic demand increases. Basically we apply this definition of network capacity to the capacity of a sightseeing area as well. However, travelers can change their route and destination due to congestion levels over the traffic network. There is a small difference that "the capacity of a sightseeing area is defined as the maximum level of entrance demand into this area such that at least one traveler is not able to reach his destination site through the road network as the total entrance demand into this area increases". When we define a sightseeing site capacity, the constraint on an environmental factor like total emission of CO and NO₂ exhausted from cars as well as physical ones used above should be introduced.

To evaluate the network capacity, there are four main methods as follows: first is the method of traffic assignment simulation, second is using cut on the Graph theory, third is an extended mathematical method of the second one using the Linear Programming method. The last one is the method used to mathematically solve a network capacity problem under network equilibrium conditions. We used the first method, the traffic assignment simulation method, because it has some merits such that, for example, we can consider traveler’s choice behavior in the assignment procedure.

In analyzing the urban traffic demand like a commute and business trip, it is appropriate to assume that OD patterns are fixed and a minimum cost route between them is chosen by trip makers. To calculate network capacity in urban area using traffic assignment simulation, the UE/PD model (User Equilibrium assignment model with Fixed Demand) is usually used. In comparison with this, visitors may change their destination according to the congestion level en route and at their original destination site. On their route choice stage, it may not always follow that they choose the minimum cost route to their destination. To evaluate the capacity of a sightseeing site using traffic assignment simulation method, the NLSUE/Dest (Nested Logit type Stochastic User Equilibrium assignment model with elastic Destination choice demand) model which can consider traveler’s stochastic and destination/route joint travel choices behavior as the assignment model of simulation is useful.
Mathematical Tool of Capacity Evaluation

The NLSUE/Dest model can describe a traveler’s multi-dimensional travel choice procedure on destination sightseeing site and route choice based on the random utility theory. In this model, a traveler assumes to choose the combination set of destination site and route which has maximum utility among his available alternatives. We specify the utility level of this combination set, \( U(od, r) \), represents as follows:

\[
U(od, r) = U(od) + U(odr) = (-C_{r}^{od} + \xi_{od}^{r}) + (V_{od} + E_{od}) \quad \forall r, od, \quad (1)
\]

where \( U(od), U(odr) \) is the random utility term by OD pair \( od \) and route \( odr \), respectively. \( C_{r}^{od} \) is the cost on route \( r \) between \( o \) and \( d \), and \( V_{od} \) is the attraction measure of \( rs \) OD pair. \( \xi_{od}^{r} \) and \( E_{od} \) are both random terms. The probability that a traveler chooses a combination of destination \( d \) and route \( r \) is written as follows,

\[
P(d, r \mid o) = P(d \mid o) \cdot P(r \mid d, o). \quad (2)
\]

Given \( \xi_{od}^{r} \) and \( E_{od} \) are dependent and identically Gumbel distributed, the expected value of the route \( r \) and OD pair \( od \) flows are given by a logit type model as

\[
f_{r}^{od} = q_{od} \cdot P(r \mid d, o) = q_{od} \cdot \frac{\exp[-\theta C_{r}^{od}]}{\sum_{r \in R_{d}} \exp[-\theta C_{r}^{od}]}, \quad \forall r, od, \quad (3)
\]

\[
q_{od} = O_{o} \cdot P(d \mid o) = O_{o} \cdot \frac{\exp[\xi(V_{od} - S_{od})]}{\sum_{d} \exp[\xi(V_{od} - S_{od})]}, \quad \forall od, \quad (4)
\]

where \( \theta \) and \( \xi \) are sensitivity parameters of route and destination choices, respectively. \( S_{od} \) is called the composite cost and is defined by

\[
S_{od} = -\frac{1}{\theta} \ln \sum_{r \in R_{od}} \exp[-\theta C_{r}^{od}] \quad \forall od. \quad (5)
\]

The relations between link cost \( t_{a} \) and route cost \( C_{o}^{r} \), and the relation between route flows \( f_{r}^{od} \) and link flows \( x_{a} \) are written as follows;

\[
C_{o}^{r} = \sum_{a} \delta_{a, t_{a}}, \quad \forall r, of, \quad (6)
\]

\[
x_{a} = \sum_{od \in R_{a}} \sum_{r \in R_{od}} \delta_{a, r} f_{r}^{od}, \quad \forall a, \quad (7)
\]

where \( \delta_{a, r} \) is the incident variable that equals 1.0 only when the route \( r \) connecting between \( o \) and \( d \) includes link \( a \).

The solutions of these non-linear equations shown by Eqs.(3), (4), (6), (7) give
the equilibrium flows of the NLSUE/Dest model. However, it is difficult to solve these equations directly. Thus we replaced them by equivalent mathematical programming as follows:

\[ \begin{align*}
\text{P0} & \quad \text{min} \ Z(f, q) = \sum_a \int_0^{x_a} f_a(x) \, dx \\
& \quad + \frac{1}{\theta} \sum r \sum_{s=d}^d q_{rd} \ln \frac{f_{rd}}{q_{rd}} + \frac{1}{\xi} \sum a \sum_{a=d}^d \frac{q_{ad}}{O_a} \ln \frac{q_{ad}}{O_a} - \sum d q_{ad} \lambda_d \\
\text{s.t.} & \quad O_a = \sum d q_{ad}, \quad \forall a \\
& \quad q_{ad} = \sum r f_{rd}, \quad \forall a \in a \\
& \quad f_{rd} \geq 0, \quad \forall r, d,
\end{align*} \]

where \( \lambda_d \) is the attraction measure of sightseeing site \( d \). It is axiomatic that the solutions of this mathematical programming problem \([\text{P0}]\) give the equilibrium flows of the NLSUE/Dest model.

The evaluation of the capacity at a sightseeing site is done by assignment simulation using Problem \([\text{P0}]\) as follows:

**Step-1**: Give an initial link flow and cost.

**Step-2**: Calculate link weight \( W^o (\forall o) \) using Dial's forward pass algorithm, and obtain the composite cost \( S_{ad} \) from the following equation.

\[ S_{ad} = C_{mn}^{ed} - \frac{1}{\theta} \ln \sum m \in I_{ad} W^m \{ m \to d \}, \quad \forall a \in a \]  

**Step-3**: Find OD flows \( q_{ad} \) from Eq.(4).

**Step-4**: Find link flows \( x_a \) using Dial's backward pass procedure.

**Step-5**: Go to Step-2 to revise link flows and OD flows using Eqs.(4) and (7).

**Step-6**: Increase total traffic demand until the limited constraint is no longer satisfied

**Parameter estimation Method of Problem \([\text{P0}]\)**

In order to evaluate the capacity using \([\text{P0}]\), we have to know the value of \( \theta \), \( \xi \) and \( \lambda_d \) in advance. We can estimate these values by solving the equivalent problem \([\text{P0}]\) as follows:

\[ \begin{align*}
\text{P1} & \quad \text{min} \ Z(f, q) = \sum_a \int_0^{x_a} f_a(x) \, dx \\
& \quad + \frac{1}{\theta} \sum r \sum_{s=d}^d q_{rd} \ln \frac{f_{rd}}{q_{rd}} + \frac{1}{\xi} \sum a \sum_{a=d}^d q_{ad} \ln \frac{q_{ad}}{O_a} - \sum d q_{ad} \lambda_d
\end{align*} \]
s.t. \( D_d = \sum \sigma q_{\sigma d}, \quad \forall d \) (14)

and Eqs.(9)-(11),

where \( D_d \) is trip attraction demand to sightseeing site \( d \). There are some differences between problems [P0] and [P1]. In problem [P0], the attraction measure \( \lambda_d \) is given and only a single constraint is included. On the other hand, in problem [P1], \( \lambda_d \) should be estimated and double constraints are included. However, the first-order conditions of problem [P1] are the same as those of [P0], because the Lagrangians of both these optimal problems becomes the same. The route and OD flows of this problem are given as

\[
f_{r}^{od} = q_{od} \frac{\exp[-\theta C_{r}^{od}]}{\sum_{r \in S_{r}} \exp[-\theta C_{r}^{od}]} , \quad \forall r, od \tag{15}
\]

\[
q_{od} = \exp\left[\left(-S_{od} + \lambda_d^*\right) \cdot \exp[\zeta (V_{od} - S_{od})]\right] \cdot \sum_{d} \exp\left[\frac{\zeta (V_{od} - S_{od})}{\sum_{d}}\right] , \quad \forall od . \tag{16}
\]

\( \lambda_d^* \) and \( \mu_d^* \) are the Lagrangian multipliers with respect to the equality constraints of Eqs.(14) and (9), respectively. Using these results, we can estimate the unknown parameters, \( \theta, \zeta \) and \( \lambda_d \) if we have observation data on OD demand between sightseeing areas.

From Eq.(15), \( \theta \), the so called route choice sensitivity parameter, can be estimated by the usual OLS. On the other hand, \( \zeta \) can be estimated by substituting Eq.(16) into Eqs.(9) and (14). Trip generation demand \( O_o \) and trip attraction demand \( D_d \) can then be rewritten, respectively, as

\[
O_o = \sum_d \exp\left[\left(-S_{od} + \lambda_d^*\right) \cdot \exp[\zeta (V_{od} - S_{od})]\right] , \quad \forall od \tag{17}
\]

\[
D_d = \sum_o \exp\left[\left(-S_{od} + \mu_d^*\right) \cdot \exp[\zeta (V_{od} - S_{od})]\right] \tag{18}
\]

When we assume \( A_o = \exp[\zeta (V_{od} - S_{od})] / O_o, B_d = \exp[\zeta (V_{od} - S_{od})] / D_d \), Eq.(16) can then be expressed as

\[
q_{od} = A_o \cdot B_d \cdot O_o \cdot D_d \cdot \exp[-\zeta S_{od} + 1] , \quad \forall od . \tag{19}
\]

\( A_o \) and \( B_d \) are well known balancing factors which are expressed

\[
A_o = \sqrt[\sum_d B_d \cdot D_d \cdot \exp[-\zeta S_{od} + 1]} , \tag{20}
\]

\[
B_d = \sqrt[\sum_o A_o \cdot O_o \cdot \exp[-\zeta S_{od} + 1]} , \tag{21}
\]

so \( \zeta \) can be estimated using the doubly constrained model represented by Eqs.(19)
through (21).

As the attraction measure \( \lambda_d \) of sightseeing area \( d \) is correspondent to the Lagrangian multiplier \( \lambda_d^* \), \( \lambda_d \) can be estimated by comparing Eqs. (16) and (19). OD flow, \( q_{cd} \), is given by

\[
q_{cd} = A_a \cdot B_d \cdot \sum_d \exp[\xi(-S_{cd} + \lambda_d^*)] \cdot \exp[\xi \mu_a^* - 1] \cdot \sum_a \exp[\xi(-S_{cd} + \mu_a^*)] \cdot \exp[\xi \lambda_d^* - 1] \cdot \exp[-\xi S_{cd} + 1].
\]  

(22)

so we rewrite the balancing factors \( A_a \) and \( B_d \) with

\[
A_a = \frac{1}{\sum_d \exp[\xi(\lambda_d^* - S_{cd})]}.
\]  

(23)

\[
B_d = \frac{1}{\sum_n \exp[\xi(\mu_n^* - S_{cd})]}.
\]  

(24)

Thus, using the value of \( B_d \) and \( \xi \) obtained by the doubly constrained model mentioned above, the attraction measure \( \lambda_d \) is given by

\[
\lambda_d = \frac{1}{\xi} (\ln B_d + \ln D_d + 1).
\]  

(25)

**Evaluation of Capacity at Aso Sightseeing area**

We applied our proposed method to evaluate the sightseeing capacity in the Aso area shown in Figure 1. The Aso area is a part of the Aso-Kuju National Park and consists of the active volcano Mt. Aso with a 20km diameter huge somma and extensive grass fields. At first, we rewrote real road network in this area into a model

![Figure 1. The Aso area with a 20km diameter huge somma and extensive grass fields](image-url)
network which consists of 53 nodes and 138 links shown in Figure 2. The national survey on sightseeing travel demand was done in 1992 by the Public Works Research Institute of the Ministry of Construction. Especially, in the Aso area, the sightseeing excursion survey was carried out in the summer vacation sightseeing season. The questionnaire sheets were distributed to 15,640 visitors who travel by car at seven sightseeing points and at facilities shown as A-A to G-G in Figure 2.

This survey was partitioned into three sections as follows;

a) visitor's individual and household socio-economic attributes
b) trip attributes from starting point to home
c) sightseeing excursion behavior inside this area.

The response rate by post was 6.1%. At the same time, traffic volume was counted by a plate-number survey at four crossings from ① to ④ as shown in Figure 2 to scale up the number of samples to population. All sites visited by respondents are aggregated into 25 locations as described by small white circles in Figure 1.

Parameters \( \theta \) and \( \xi \) were estimated 0.020 and -0.0077, respectively by using our parameter estimation method mentioned above. The estimation results of attraction measures \( \lambda_d \) at each sightseeing site \( d \) are shown in Table 1. \( \lambda_d \) estimated has the same unit of travel cost. We substituted these estimation values into [P0] and tried to solve the problem. The relation between observation and forecasting values on link flow is shown in Figure 3. The correlation coefficient is 0.82 and the F-value is 1071.8. These values are high and problem [P0] can sufficiently function to describe the real traffic demand, so it is applicable as an assignment simulation model to evaluate sightseeing capacity of the Aso area.

Resulting from the NLSUE/Dest assignment simulation, we found that the total
Table 1  Attraction Measures of Sightseeing Site

<table>
<thead>
<tr>
<th>site</th>
<th>$\lambda_i$</th>
<th>site</th>
<th>$\lambda_i$</th>
<th>site</th>
<th>$\lambda_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oguni</td>
<td>704.5</td>
<td>Akamizu Spa</td>
<td>690.7</td>
<td>Mt. Aso</td>
<td>317.7</td>
</tr>
<tr>
<td>Kurokawa Spa.</td>
<td>551.7</td>
<td>Cuddly Dominion.</td>
<td>446.7</td>
<td>South Aso Resort</td>
<td>559.1</td>
</tr>
<tr>
<td>Sanai Restaurant</td>
<td>501.1</td>
<td>Aso Station</td>
<td>647.9</td>
<td>Tarutama Spa.</td>
<td>650.6</td>
</tr>
<tr>
<td>Aso Skyline</td>
<td>685.2</td>
<td>Aso-shrine</td>
<td>656.4</td>
<td>Tawarayama</td>
<td>672.7</td>
</tr>
<tr>
<td>Dui-Kambou</td>
<td>405.9</td>
<td>Monkey theater</td>
<td>462.5</td>
<td>Greenpia south Aso</td>
<td>570.1</td>
</tr>
<tr>
<td>Elpacio farm</td>
<td>643.5</td>
<td>Kusasenri</td>
<td>292.5</td>
<td>Shirakawa</td>
<td>490.2</td>
</tr>
<tr>
<td>Ubuyama</td>
<td>760.7</td>
<td>Sensui Vallay</td>
<td>606.8</td>
<td>Takamori</td>
<td>668.2</td>
</tr>
<tr>
<td>Kabutoiwa</td>
<td>777.1</td>
<td>Tochinoki Spa.</td>
<td>601.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uchinomaki Spa.</td>
<td>555.2</td>
<td>Yunotani Spa.</td>
<td>608.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Trip demand in this area is 70,549 trips. Then, the sightseeing capacity of the Aso area, that is the maximum entrance demand, becomes 24,248 trips. This value seems to be very reasonable because there were observation results that about 30,000 people visited this area in the highest sightseeing season.

Conclusion

In this paper, the capacity of a sightseeing area is defined as the maximum level of entrance demand by cars that at least one traveler is not able to reach his destination as total entrance demand into the area increases. A mathematical model based on network equilibrium theory was proposed to evaluate this capacity and to estimate its unknown parameters. Finally, this model was applied to the Aso national park area to evaluate the sightseeing capacity of the Aso area. The capacity value evaluated was reasonable based on results of observations, verifying the applicability of our proposed method.

References