Dynamic Dispersion Curves for Pipes on Elastic Foundation

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The problem of wave propagation in pipe on foundation is very important in both civil and mechanical engineering fields. This paper presents a complex field general variational statement to study dynamic dispersion curves of waves propagating in pipe-foundation system. A semi-analytical technique is used to construct the variational complex wave field and a three dimensional continuous medium model is used for its general applicability. The formulation and its program are verified by comparing with the available results obtained using other methods for a freely supported solid elliptical cylinder. Numerical results show dispersion changes of varying degree in the presence of foundation and reveal significant influences of foundation on low order propagating modes and low frequency range. The results are further discussed in details for different fields of practical use and frequency ranges of engineering interest.

Keywords: elastic foundation; wave propagation; dispersion curves; pipe

1. INTRODUCTION

In both civil and mechanical engineering fields, pipe structures on elastic foundation can be found in many situations, for examples, a shaft within a hydrodynamically lubricated bearing 1), a pipe buried in soil, etc. The pipe on foundation is distinguished from the foundation by its waveguide geometry and usually a difference in impedance. Mechanical vibration in the pipe, generated by seismic excitation, machine, traffic, etc., can transmit for long distances by using series of reflections between the cross section boundaries of the pipe to its propagation advantage, which can be exploited for good uses, e.g., signal transmission and non-destructive testing, but also can create problems, e.g., distress to pipe structures, adjacent machines and annoyance to peoples.

Vibration is commonly described in terms of modes or wave motions. For a pipe-like structure that behaves as if it has infinite length or where high frequency and short wavelength motions are of concern, using a wave model is more appropriate than using a modal model to describe the dynamic behavior of the structure 2,3) as the modes of vibration are standing waves by nature and result from interferences of the travelling waves. This study is one in a series study that intends to control propagating or energy-carrying vibrations in pipe structures on foundation by exploiting the system’s inherent properties of vibration propagation. It is motivated by the work of wave filtering design for optical, electronic and acoustic devices, in which the material or structure is designed to have frequency pass and stop bands desired by using dispersion relationships of waves in systems, e.g., refs. 4-5).

This paper focus on computing dynamic dispersion curves in the pipe-foundation system. Obtaining the dispersion curves and understanding the wave propagation information in systems are, in fact, important in various applications, e.g., in studying near field or high frequency motions; in developing an effective method of non-destructive evaluation and noise control; in interpretation of earthquake records; etc.4,9) Take the non-destructive testing as an example. The dispersion curves are basis for all physically based guided-wave testing and data interpretation 9). However, for structures on foundation, no related work has been found in literature, while there is a large number of published paper studying static and dynamic problems of structures on elastic foundation as well as wave propagation problems of freely supported structures. These papers include those that are very recent, for examples, refs. 10-11).

There are many models for elastic foundation, but the one used most frequently could be the Winkler theory of equivalent spring, because of its apparent simplicity and ability to describe many engineering problems. This paper presents a complex field variational statement to study the dynamic dispersion curves for pipes on elastic Winkler foundation. A semi-analytical technique is used to construct the variational complex wave field for the variational statement. The semi-analytical technique is based on factorization of the function describing the complex displacement field. The cross section is modeled in a manner analogous to
conventional finite element method while the displacement along wave propagation direction is expressed analytically in the form of harmonic plane wave.

Available work that are relevant to the similar technique and deal with various problems can be found in refs. 12-13, where buried structures subjected to three-dimensional surface loading are analyzed both statically and dynamically; and refs. 14-15, where they present the technique for analysis of piezoelectric structures under mechanical and electrical loads. One should note that the term “semi-analytical” is for methods in which the total solution is obtained by the simultaneous and consistent use of an analytical and a numerical method, as noted in ref. 15, and the numerical method is not limited to the finite element method used in this paper; for example, a meshfree numerical method is used in ref. 15 in the application of the semi-analytical technique to Almansi-Michell problems of piezoelectric cylinders.

This paper’s outline is as follows. Sections 2 and 3 develop the complex field variational statement and its semi-analytical solution. The results, including verification, numerical examples and the detailed result discussion, are presented in Section 4. Finally, summary and conclusions are given in Section 5.

2. COMPLEX FIELD VARIATIONAL STATEMENT FOR HARMONIC MOTIONS OF PLANE WAVE

Consider the spatial configuration of a general deformable body defined by domain \( \Omega \subset \mathbb{R}^3 \) which is described by Cartesian coordinates \((x, y, z) := x\) with the hybrid boundary condition on Winkler foundation supported area \( \partial \Omega \). Let \( n_f \) be the unit normal vector to \( \partial \Omega \), \( u, C, \sigma \) and \( e \) be the displacement vector, forth order elastic tensor, second order stress and strain tensors respectively, with their matrix forms:

\[
\begin{align*}
[n_f] &= [n_x, \ n_y, \ n_z]^T, \ [u] = [u_x, \ u_y, \ u_z]^T, \\
[e] &= [e_{xx}, \ e_{yy}, \ e_{zz}, \ 2e_{xy}, \ 2e_{xz}, \ 2e_{yz}]^T, \\
[\sigma] &= [\sigma_{xx}, \ \sigma_{yy}, \ \sigma_{zz}, \ \sigma_{xy}, \ \sigma_{xz}, \ \sigma_{yz}]^T
\end{align*}
\]  

for which the following relationships hold:

\[
\nabla = \left[\begin{array}{cccccc}
\frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial z} \\
0 & \frac{\partial}{\partial y} & 0 & 0 & \frac{\partial}{\partial z} \\
0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & 0 \\
\end{array}\right], \quad \frac{\partial}{\partial z} = \left[\begin{array}{c}
\frac{\partial}{\partial x} \\
\frac{\partial}{\partial y} \\
\frac{\partial}{\partial z}
\end{array}\right] \left[\begin{array}{c}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{xy} \\
\sigma_{xz} \\
\sigma_{yz}
\end{array}\right] = \\
\nabla u = \sigma \varepsilon = \varepsilon_{ij} \frac{\partial}{\partial x_i} u_j
\]

For harmonic waves with frequency \( \omega \), the common complex formalism should be adopted\(^1\), and the displacement, strain, and stress are written as:

\[
[\sigma, e, u] = \Re \left[ [\sigma, e, u](x, \omega) \exp(-i\omega t) \right]
\]

where \( i = \sqrt{-1} \), and \( \bar{u}, \bar{e}, \bar{\sigma} \) are the amplitudes. So the kinetic energy of the structure \( \Omega \) is

\[
T = \frac{1}{2} \iiint_{\Omega} \rho \omega^2 \left( \bar{u}^T \varepsilon \bar{u} \right) \, d\Omega
\]

where \( (\ast) \) denotes the complex conjugate and \( \rho \) is the density. In the absence of body forces and external loads, the potential energy of structure \( \Omega \) is

\[
V = \frac{1}{2} \iiint_{\Omega} (C \varepsilon)^T \varepsilon \, d\Omega
\]

and the potential energy of Winkler foundation is

\[
V_{wf} = \frac{1}{2} \iint_{\partial \Omega} k_f \left[ \bar{n} \right]^T \left[ n_f \right] \, dS
\]

where \( k_f \) is the Winkler spring stiffness. Since Hamilton’s principle is in the form:

\[
\delta \left( T - V \right) \, dt = 0
\]

and note that \( T \) and \( V = V_{wf} + V \) is independent of time \( t \) here, we can obtain the variational statement as:

\[
\delta \delta T = \iiint_{\Omega} \rho \omega^2 \left( \bar{u}^T \varepsilon \bar{u} \right) \, d\Omega - \iiint_{\Omega} (C \varepsilon)^T \varepsilon \, d\Omega - \iint_{\partial \Omega} k_f \left[ \bar{n} \right] \left[ n_f \right] \, dS + c.c. (8)
\]

\[
= 0
\]

where c.c. denotes the complex conjugate of the quantities which precede it.

Fig. 1 Schematic of a pipe structure on elastic foundation: a pipe buried underground.

Let us consider a pipe structure \( \Omega \) embedded in the Winkler foundation and the harmonic waves propagating along
the \( z \) axis direction as shown in Fig. 1. For the Cartesian coordinates used to describe the system, the waves in the system are of the following plane wave form:

\[
\bar{u}(x, \omega) = [\hat{u}(x, y, \omega, k)] \exp(ik_z z)
\]

(9)

where \( \hat{u} \) is amplitude and \( k_z \) is wave number that describes the dispersive properties of system and is defined in terms of phase velocity \( c_p(\omega) \) and attenuation coefficient \( \alpha(\omega) \):

\[
k_z = i\alpha(\omega) + \frac{\omega}{c_p(\omega)}
\]

(10)

The wave number can be purely real, imaginary or complex, associated with propagating, evanescent or decaying oscillating wave, respectively. If \( \Re(k_z(\omega)) \) is expanded in a Taylor series, the first-order expansion coefficient defines the group velocity \( \omega_c(\omega) \):

\[
\omega_c(\omega) = \frac{1}{c_p(\omega)} \frac{dc_p(\omega)}{d\omega}
\]

(11)

It follows that:

\[
c_p(\omega) = \omega / \omega_c(\omega)
\]

(12)

Under normal dispersion conditions, which are in contrast to the abnormal conditions when the group velocity exceeds the phase velocity or becomes less than zero, the group velocity is the velocity of the wave package or the velocity at which energy is conveyed along the wave.

Substituting Eq. (9) into Eq. (8), and using the obvious facts that \( \exp(i k_z z) \) \( \exp(i k_z z) \) = 1 and the equation holds for arbitrary \( z \) interval, give the following integration domain reduced equation

\[
\delta \hat{I}(\bar{u}) = \int_{S_p} -\nabla \cdot \delta \hat{u}^{\ast} \cdot \delta \bar{u} \cdot [C] \cdot \nabla \bar{u} + ik \nabla \cdot \delta \hat{u}^{\ast} \cdot [C] \cdot \hat{u} \cdot \nabla \bar{u} + \int_{S_p} \rho \omega^2 \delta \hat{u}^{\ast} \cdot [\hat{u}] \cdot \nabla \bar{u} \cdot ds_p
\]

+ \int_{L_{w}} k_j \cdot [n] \cdot [n] \cdot \nabla \bar{u} \cdot dl_{w} + c.c.
\]

(13)

in which

\[
\nabla_z = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\nabla_S = \begin{bmatrix}
\frac{\partial}{\partial x} & 0 & 0 & 0 & 0 & \frac{\partial}{\partial y} & 0 \\
0 & \frac{\partial}{\partial y} & 0 & 0 & 0 & \frac{\partial}{\partial x} & 0 \\
0 & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 & 0 & 0 \\
\end{bmatrix}
\]

(14)

and \( S_{w} \) is the cross section area of the pipe, \( L_{w} \) is the corresponding Winkler foundation supported edge of \( S_{w} \) as shown in Fig. 1.

3. SEMI-ANALYTICAL FINITE ELEMENT METHOD SOLUTION

The pipe cross section \( S_{w} \) and its corresponding Winkler foundation edge \( L_{w} \) can be represented by a system of finite elements with domains \( S^e \) and \( L^e \) respectively, while let the wave motion along the \( z \) axis direction in Eq. (9) remains unchanged so that \( z \) axis motion can be expressed analytically. In a typical element, the complex displacement field can be written in terms of the shape functions, \( [N] \) and the nodal unknown displacements \( [U] \), thus the vector \( \{\hat{u}(x, y)\} \) in Eq. (9) can be written as:

\[
\{\hat{u}(x, y, \omega, k)\}^T = [N(x, y)]^T \{U(\omega, k)\}
\]

(15)

where the superscript \((\cdot)^T\) denotes an element level matrix or vector. The substitution of Eq. (15) into Eq.(13), followed by algebraic manipulations, yields

\[
\delta \hat{I}(\bar{u}) = \sum_{j=1}^{ne} \int_{S^e} -\nabla \cdot [N]^T \cdot [C] \cdot [N] \cdot \{U\}^T + ik \nabla \cdot [N]^T \cdot [C] \cdot \{U\} \cdot \nabla \{U\}^T + \rho \omega^2 [N]^T \cdot [N] \cdot \{U\} \cdot \nabla \{U\}^T + \sum_{j=1}^{nef} \int_{e^e} \delta [U]^T \cdot \{U\} \cdot \nabla \{U\}^T + c.c.
\]

(16)

where \( ne \) and \( nef \) are the total numbers of elements in their corresponding domains. The number of elements and order of shape function should be consistent in the shared edge of the Winkler foundation and pipe cross section.

For the arbitrary variation of the indicated quantities \( \delta [U]^T \), the above equation yields the following homogenous equation with each term at its global representation:

\[
\sum_{j=1}^{ne} [K]^T \cdot [U] + \sum_{j=1}^{nef} [K]^T \cdot [U] - \omega^2 \sum_{j=1}^{ne} [M]^T \cdot [U] = 0
\]

(17)

where

\[
[M] = \sum_{S} \rho \cdot [N]^T \cdot [N] \cdot dx dy
\]

\[
[B] = \nabla_z \cdot [N] + ik \cdot \nabla_s \cdot [N]
\]

\[
[K]^T = \sum_{S} [B]^T \cdot [C]^T \cdot [B] \cdot dx dy
\]

(18)

\[
[K]^T = \int_{e^e} k_j \cdot [N]^T \cdot [n] \cdot [n] \cdot [N] \cdot d\ell
\]

By defining set \( \{(\xi, \eta) \in \mathbb{R}^2 | [-1,1]\} \) for an isoparametric
element, and maps:

\[ f_i : \mathbb{R}^1 \rightarrow \mathbb{R}^2, \quad f_i : (\xi) \mapsto \left( [N(\xi)]^T [X]^T, [N(\xi)]^T [Y]^T \right) \in \mathbb{L} \]

\[ f_i : \mathbb{R}^2 \rightarrow \mathbb{R}^2, \quad f_i : (\xi, \eta) \mapsto \left( [N(\xi,\eta)]^T [X]^T, [N(\xi,\eta)]^T [Y]^T \right) \in \mathbb{L}^* \]  

(19)

where \( [X]^T, [Y]^T \) are the coordinates of element nodes, \( [N(\xi,\eta)]^T \) is the same shape function as in Eq. (15), and \( [N(\xi)]^T \) is the one dimensional standard shape function of the same order as \( [N(\xi,\eta)]^T \), the entries in Eq. (17) can be expressed as:

\[ [K]^T = \int_{\Omega} \int_{\Gamma} [B]^T [C^*] [B] \text{det}(J_{\beta}(\xi,\eta)) d\xi d\eta \]

\[ [M]^T = \int_{\Omega} \int_{\Gamma} \rho[N]^T [N]^T \text{det}(J_{\beta}(\xi,\eta)) d\xi d\eta \]

(20)

\[ [K]^T = \int_{\Omega} \int_{\Gamma} k_j [N]^T [n_j]^T [N]^T \left| J_{\beta}(d\xi) \right| \]

where \( \text{det}(J_{\beta}(\xi,\eta)), j = 1, 2 \) is the determinant of Jacobian matrix \( J_{\beta}(\xi,\eta) \) of mapping \( f_i \), and \( \| \bullet \| \) is the norm of vector \( \bullet \). The unit normal vector \( [n_j] \) in Eq. (20) can be obtained by mapping from the vector that is made by rotating 90 degree the unit vector along \( \xi \) direction in parameter element:

\[ n_j = \frac{R_{\beta}(1)}{\| R_{\beta}(1) \|}, \quad R = \begin{bmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{bmatrix} \]  

(21)

The standard finite element assembling procedures can be used here to rearrange Eq. (17) into the following generalized eigenvalue problem (EVP):

\[ Ax = \lambda Bx \]  

(22)

in which

\[ x = \sum_{j=1}^{n} [U]^T, \quad \lambda = \omega^2, \quad B = B^T = \sum_{j=1}^{n} [M]^T, \]

(23)

\[ A = A^T = \sum_{j=1}^{n} [K]^T + \sum_{j=1}^{n} [K]^T \]

where \( x \) is the global vector of the unknown nodal displacements, \( B \) is a real-valued symmetric matrix and \( A \) is a Hermitian matrix. Note that in the present case, all eigenvalues are real, and all eigenvectors satisfy orthogonality conditions.

Since only propagating waves are of interest in this paper, the EVP is solved for given values of real valued wave number \( \Re \{k_j\} \). Thus, pairs \( \Re \{k_j\}, \omega, c_1, c_2 \) and their eigenvectors give all propagating wave information in the system. However, we should note that the group velocity computed from its definition in Eq.(12) requires the differentiation of discrete frequencies and wave numbers, and the discrete differentiation raises numerical challenges to get accurate group velocity and convert the discrete solutions into different continuous branches of modes. This can be solved by following the approach used in ref. 18. First we evaluate the derivative of Eq. (22) with respect to the wave number:

\[ \frac{\partial A}{\partial k_z} - \frac{\partial A}{\partial k_z} Bx + (A - \lambda B) \frac{\partial x}{\partial k_z} = 0 \]  

(24)

Note that, alternatively Eq. (22) can also be written in term of left eigenvector \( x^l \) as:

\[ x^l A = \lambda x^l B \]  

(25)

and for a complex Hermitian EVP, the following relations hold:

\[ x^l = x^*, \quad \lambda^l = \lambda \]  

(26)

Then, multiply Eq. (24) from the left by the left eigenvector \( x^l \), the following can be obtained:

\[ \frac{\partial \lambda}{\partial k_z} = x^l \frac{\partial A}{\partial k_z} x^l / 2 \omega x^l Bx \]  

(27)

Therefore,

\[ c_g = \frac{\partial \omega}{\partial k_z} = \frac{\partial \omega}{\partial \lambda} \frac{\partial \lambda}{\partial k_z} = x^l \frac{\partial A}{\partial k_z} x^l / 2 \omega x^l Bx \]  

(28)

In Eq. (27) and Eq. (28),

\[ \frac{\partial A}{\partial k_z} = \sum_{j=1}^{\infty} \left[ -iN_N^T [N]^T [C^*] [B] \right] dx \]

\[ + \sum_{j=1}^{\infty} \left[ (B)^T [C^*] (i+N_N^T [N]^T) \right] dx \]  

(29)

From Eq. (28) the group velocity can be solved without requiring the discrete differentiation.

In this paper, a C++ program based on the framework of OOFEM Lib 19) is written to form Eq. (22), and ARPACK++, 20) which is an object-oriented version of the well known ARPACK 20) package, is used in the program to solve Eq. (22) efficiently. In the program, a second order 8-node 2D element in 3D space for the pipe cross section and the corresponding 3-node 1D element in 3D space for the Winkler foundation are developed. They are general elements based on the continuous medium model, and can catch all propagating modes in the pipe-Winkler foundation system, provided the mesh of the cross section of pipe and Winkler foundation is fine enough. This is important part of developing an effective non-destructive evaluation and noise control method, where high frequency waves are of concern.

4 VERIFICATION AND RESULTS

The pipe-foundation system has a symmetric geometry with respect to the horizontal and vertical mid-plane as denoted in Fig. 1. For isotropic pipes, this advantage can be taken to de-couple symmetric motions from antisymmetry motions; therefore, only one quarter of the system’s cross section requires discretization.

There is an inherent requirement that the elements
representing the cross section of the pipe and the supporting foundation should have adequate refinement to catch the highest propagating mode of interest. Here the mesh refinement used in all of examples studied is determined simply by convergence studies for the modes of interest.

4.1 Verification

A solid elliptical cylinder of ellipticity 0.5 and Poisson’s ratio 0.3 in vacuum was studied by Kynch and Green (22), and Fraser (23) using perturbation and collocation methods respectively. So an important test is to check whether our program can obtain the same result. Comparisons with their results are made in Table 1 and Fig. 2, where the agreements can be seen. In Table 1, the perturbation results are converted from ref. 23) by Fraser, and $b$ is the minor semi-axis length of the elliptical cylinder.

![Fig. 2 Comparison of the nondimensionalized results with the published by Fraser in ref. 23) for the extension modes of a solid elliptical cylinder in vacuum. ○ Fraser, — present solution.](image)

Table 1 Comparison between results obtained using different methods for the 1st extension mode of a solid elliptical cylinder of ellipticity 0.5 and Poisson’s ratio 0.3.

<table>
<thead>
<tr>
<th>$k_b$</th>
<th>Perturbation</th>
<th>Collocation</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3901</td>
<td>1.6058</td>
<td>1.6058</td>
<td>1.6058</td>
</tr>
<tr>
<td>1.2419</td>
<td>1.5212</td>
<td>1.5211</td>
<td>1.5211</td>
</tr>
<tr>
<td>1.7095</td>
<td>1.4193</td>
<td>1.3503</td>
<td>1.3503</td>
</tr>
<tr>
<td>3.9462</td>
<td>1.0151</td>
<td>0.9905</td>
<td>0.9905</td>
</tr>
<tr>
<td>5.0000</td>
<td>——</td>
<td>0.9596</td>
<td>0.9597</td>
</tr>
</tbody>
</table>

Compared to the method used in this paper, the perturbation method becomes complicated at points where mode branches intersect and at such points it gives less accurate results (e.g., when $k_b = 1.7095$ in Table 1), while the collocation method suffers from the disadvantages that it is not suitable for cross sections such as very narrow ellipses and rectangles or cross sections with re-entrant corners such as an L-shaped cross section, as also noted in ref. 23).

4.2 Numerical results

In this section, a circular pipe embedded in elastic foundation that corresponds to a kind of coarse grained sand backfill in engineering, with the following geometry and physical property, is given as an illustration example:

$$E = 3.1 \times 10^9 \text{N/m}^2, \quad \rho = 2300 \text{kg/m}^3, \quad \nu = 0.25,$$

$$k_r = 0.5 \times 10^9 \text{N/m}^3, \quad R = 0.3 \text{m}, \quad h = 0.05 \text{m}$$  \(30\)

where $h$ and $R$ are the thickness and outer radius of pipe respectively; $E$ and $\nu$ are Young’s elastic modulus and Poisson’s ratio of the isotropic pipe respectively.

![Fig. 3 Dispersion curves of propagating waves: (a) frequency vs. wave number relationship; (b) phase velocity vs. frequency relationship; (c) group velocity vs. frequency relationship.](image)
computed from Eq. (28). These dispersion curves are very important for practical uses, e.g., they are basis for all physically based guided-wave testing and data interpretation as mentioned before. Each point on the curve corresponds to a resonance pattern of guided-wave propagation. In addition, for each point, there is a unique distribution of displacement field and stress field. To assist understanding, the corresponding freely supported pipe is also analyzed and shown in Fig. 3 for comparison purposes.

![Image 1](image1.png)

**Fig. 4** One-quarter of the typical extensional mode motions (symmetry about both the symmetric planes): (a) $E1$, (b) $E2$.

![Image 2](image2.png)

**Fig. 5** One-quarter of the typical torsional mode motions (antisymmetry about both symmetric planes): (a) $T1$, (b) $T2$.

![Image 3](image3.png)

**Fig. 6** One-quarter of the typical flexural mode motions (symmetry about one of the symmetric plane and antisymmetry about the other one symmetric plane): (a) $Fx1$, (b) $Fx2$.

Various branches are classified and denoted according to their predominant motions and orders of mode shape. Predominant motion of extension, torsion, or flexures about the two symmetric axes is denoted by letters $E$, $T$, $Fx$ or $Fy$; their typical mode shapes up to the second lowest-order are given in the form of 3D displacement field in Figs. 4-6 respectively, which provide some insight into the distribution of energy across the cross section of the pipe-foundation system for their corresponding mode branches.

The fact that the isotropic pipe is of circle shape and fully embedded in foundation (thus, infinitely many planes of symmetry) makes the dispersion curves of modes $Fx$ and $E2$ coincide with those for the modes $Fy$ and $T2$ as shown in Fig. 3. Some insight into these can be gained from the mode shapes of modes $E2$ and $T2$ shown in Fig. 4b and Fig. 5b respectively: they are same if we reflect the one quarter models into the full ones according to their symmetry or anti-symmetry conditions at the symmetric boundaries.

Compared to the freely supported pipe where the flexural mode has solution that extends to the origin of the frequency axis, no solution for flexural modes exists in the presence of foundation below cutoff frequencies. This is result of real wave number that is associated with the propagating waves. For example, below the cutoff frequency, solutions of complex wave number and non-propagation wave can exist for mode $Fx1$.

### 4.3 Result discussions

In Fig. 3, we can see that the presence of foundation causes the dispersion relationship changes of varying degree that depend on mode shape, frequency, and branch. However, as can be expected, because the Winkler foundation is decoupled from the structure and takes only the displacements normal to the outer surface into account, only those propagating modes that have displacement component in the normal direction are affected; for example, as can be seen in Fig. 3b, the relatively non-dispersive modes $E1$ and $T1$ (straight lines) in the case of pipe on foundation remain the same as in the case of freely supported pipe; by observing Fig. 4a and Fig. 5a, we know that this can be explained by the predominantly longitudinal and torsional motions associated with the $E1$ and $T1$ modes respectively.

In general, those foundation affected branches in the presence of foundation are cut off at higher frequency, as can be readily observed from the frequency axes in Fig. 3. These higher cutoff frequencies extend their influences on their corresponding branches but only to the low frequency ranges; for example, as can be seen in Fig. 3c, the $Fx1$ and $Fy1$ mode branch curves in the case of pipe on foundation shift rightward, but in the high frequency range they converge to their counterparts in the case of freely supported pipe. Compare the mode branches of different order with each other, more significant influences of foundation can be observed on the low order branches; for example in Fig. 3c the cutoff frequency of the 1st modes $Fx1$ and $Fy1$ in the presence of foundation is about 20 Hz larger than the ones of the 2nd modes $Fx2$ and $Fy2$, and their branch curves shift more rightward in the low frequency range.

For most engineering problems, e.g. earthquakes, low frequency and order modes tend to dominate the motion in the system; therefore, the dynamic responses in the system are significantly affected by the foundation. In earthquake engineering, structure damages are often related to the linear amplification of seismic waves passing through soft soil layers, which is correct for small amplitudes of motion or small
strains in the soil. From the wave propagation point of view in the Winkler model of foundation, this can be explained by the passing effect of the soft site and filtering or suppressing effect of the stiffer site for destructive energy-carrying modes.

In contrast, for the problems where high frequency waves are of concern, e.g., non-destructive testing, the foundation appears to have no effect at present example in Fig. 3, and the testing scheme devised based on dispersion curves of a freely supported pipe is applicable. However, if one has the close-up at the high frequency range of the dispersion curves, it can be known that the effect can still be noticeable for non-destructive testing although that depends on tested distances. For example, as denoted by the oval in Fig.3c, the velocity difference of 1.9 m/s can provide enough rich source of information for characterizing the type of case and thus sizing defects, locating defects, etc., in each of the cases, which can be achieved through either the frequency based testing scheme with the controlled exciting source or the phase velocity based testing scheme with the controlled geometry of the transducer according to Snell’s law. In addition, in mechanical engineering fields, the pipe-foundation system usually can have a higher ratio of foundation to pipe stiffness; therefore, the foundation influences will be larger on the high frequency range of dispersion curves.

The features of the propagating waves can also be exploited for uses in many ways, one good example of which is in ref. 7) where much information of waves excitable by a high-speed train viaduct and in the bellowing soil are extracted from the dispersion curves, and then a honeycomb wave impeding barrier is designed theoretically to modulate the waves and prevent undesirable waves from propagating through the soil into nearby buildings. However, one should note that, in ref. 7), the structure is regarded as some kinds of general vibration source while the surrounding as the only propagation medium. In this paper, both the structure and its surrounding are studied as the propagation medium and the waves can propagate with more vibration energy through the supported pipe structure as well. Another example is as in the work of deigning optical, electronic and acoustic devices, e.g., refs. 4-5), the pipe can be designed to have desirable pass bands and stop bands in combination with its supporting foundation design by using the cutoff features of the propagating waves in the system. This idea is applied on a simple plate-foundation system in ref. 26), where 32Hz vibration waves were effectively reduced and even suppressed on the system without the need to invoke additional contributions from the material damping by a simple barrier design that utilizes the inherent property of the propagating wave in the system.

5. SUMMARY AND CONCLUSIONS

We present a complex field variational statement and its semi-analytical solution to study the problem of wave propagation in pipes on elastic foundation. The continuous medium model is used as it has the ability to catch all propagating modes in system. Computer program based on the well known ARPACK package is written as well. The method and program are verified by comparing with the published results for a solid elliptical cylinder in vacuum.

The results are given in the forms of wave number, phase velocity and group velocity dispersion curves for the eight lowest-order positive going waves in the frequency range studied. The comparisons of results between cases of freely supported pipe and pipe on foundation show that the presence of foundation causes the dynamic dispersion relationship changes of varying degree that depend on mode shape, frequency, and dispersion branch; and the practical use of the results should takes the field of application into account.

Finally, one should keep in mind that the results are subjected to the approximation natures of the finite elements for the system’s cross-section and Winkler model for the pipe-foundation interface mechanism. Especially, Winkler foundation cannot model the surface wave of Rayleigh type along the pipe’s radius direction when the foundation has infinity stiffness; and the analysis requires the pipe-foundation system has a uniform physical and geometric property along its propagating axis. Nevertheless, representing the foundation’s ability by the single parameter of Winkler model provides a reliable and convenient way in many practices; and the analysis can allow modeling for cross-section of heterogeneous pipe material and non-uniform foundation.

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